BERGEN UNIVERSITY COLLEGE
Faculty of Engineering

EXAMINATION IN : Dynamics
CLASS : TOM 027
DATE : December 5, 2013

NUMBER OF TASKS : FOUR
NUMBER OF PAGES : 6 (INCLUDING COVER SHEET)
APPENDIXES : ALREAD ATTACHED

EXAM AIDS : SIMPLE CALCULATORS
NO DISPLAYS OF TEXT FILES

TIME : THREE HOURS
LANGUAGE : ENGLISH

ACADEMIC RESPONSIBLE
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EXAMINER(S) : 955 57 006

NOTES :

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**PROBLEM NUMBER (14.97) 1: 20 points**

The 75-kg man bungee jumps off the bridge at A with an initial downward speed of 1.5 m/s.

He is attached to the bridge by an elastic cord with stiffness \( k = 3 \text{kN/m} \)

He stops momentarily, exactly at the surface of the water.

You may neglect the size of the man.

Determine the required unstretched length of the elastic cord to which he is attached:

1. State the law (5 points)
2. Fill in all values (5 points)
3. Solve for the unstretched length of the cable (10 points)

**PROBLEM NUMBER 2 (In text sample problem): 20 points**

A 600 Kg cannon fires a 4Kg shell.

The shell leaves at a speed of 450 m/sec

The firing happens in 0.03 sec

During this time, the ejection force is a constant \( F \), as shown: \( F \) by the cannon on the shell, \( -F \) by the shell on the cannon.

What is the cannon recoil velocity?

What is the average force, \( F \) during this time?

1. State the equation or equations needed to find the recoil velocity (5 points)
2. Solve for the recoil velocity of the cannon (5 points)
3. State the equation needed to solve for the force \( F \) (5 points)
4. Solve for the force \( F \) (5 points)
PROBLEM NUMBER 3 (16.118): 30 points

The hydraulic cylinder \( D \) extends with a velocity of \( 4m/\text{sec} \) and an acceleration of \( 1.5m/\text{sec}^2 \)

The figure states FEET but use METERS

1. Determine the horizontal and vertical velocity of point \( C \)  
   (5 points)
2. Determine the angular velocity of link CBA  
   (5 points)
3. Determine the horizontal and vertical acceleration of point \( C \)  
   (10 points)
4. Determine the angular acceleration of link CBA  
   (10 points)

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PROBLEM NUMBER 4 (17.105): 30 points

The spool has a mass of 100 kg.

The spool has a moment of inertia about \( G \):  
\[ I_G = 10 \text{kg} \cdot \text{m}^2 \]

The coefficients of static friction between the spool and ground at \( A \) is: \( \mu_s = 0.2 \)

The coefficients of kinetic friction between the spool and ground at \( A \) is: \( \mu_k = 0.15 \)

A cord wraps around the inner part of the spool.

The cord is pulled with a force \( P = 600\text{N} \)

1. State all equations and draw the free body diagram  
   (5 points)
2. State the assumption  
   (5 points)
3. Find the acceleration, angular acceleration, friction and normal force  
   (10 points)
4. Test the assumption and resolve if necessary  
   (10 points)
1. Kinematics \[ a = \frac{dv}{dt} \quad v = \frac{ds}{dt} \quad adt = v dv \]

2. Constant Acceleration
\[ v = v_0 + a_t t \quad s = s_0 + v_0 t + \frac{1}{2} a_t t^2 \quad v^2 = v_0^2 + 2a_t (s - s_0) \]

3. Projectiles (where we assume “+y” is up-vertical – as is gravity; and “x” is horizontal)
\[ v = (v_0)_x \quad x = x_0 + (v_0)_x t \quad (v_x)^2 = (v_0)^2 \]
\[ v = v_0 - gt \quad y = y_0 + (v_0)_y t - \frac{1}{2} gt^2 \quad (v_y)^2 = (v_0)_y^2 - 2g (y - y_0) \]

4. NT:
Begin with: \[ v = v u_t \]
Derive-1 \[ a = a_t u_t + a_n u_n \quad \text{Where:} \quad a_t = \dot{v} \quad a_n = \frac{v^2}{\rho} \]

5. R:
Begin with: \[ r = r u_t \]
Derive-1: \[ v = v_r u_r + v_0 u_{\theta} \quad \text{Where:} \quad v_r = \dot{r} \quad v_0 = r \dot{\theta} \]
Derive-2: \[ a = a_r u_r + a_{\theta} u_{\theta} \quad \text{Where:} \quad a_r = (\ddot{r} - r \dot{\theta}^2) \quad a_{\theta} = (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \]

6. Relative \[ r_B = r_A + r_{B/A} \quad v_B = v_A + v_{B/A} \]
\[ a_B = a_A + a_{B/A} \]

7. Forces: In general \[ \sum F = ma \]
Cartesian: \[ \sum F_x = ma_x \quad \sum F_y = ma_y \]
NT: \[ \sum F_n = ma_n \quad \sum F_t = ma_t \]
R:\[ \sum F_r = ma_r \quad \sum F_{\theta} = ma_{\theta} \]

8. Formula \( \psi \) is a positive measured from extended radial line to tangent in counterclockwise direction: \[ \tan \psi = r / (dr / d\theta) \]

9. Center of mass \( r_{cm} = \sum m_i r_i \]

10. Curvature Formula \( \rho = \frac{1 + (dy/dx)^2}{}^{1/2} \)

12. Sector: \( s = r \theta \) (and for constant radius): \( \dot{s} = r \dot{\theta} \)

13. Springs \( F = k \times \text{extension} \)

14. Work: \( \int F \cdot ds \)

15. Kinetic Energy: \( \int ma \cdot ds = \int mv \cdot dv = \frac{1}{2} mv^2 - \frac{1}{2} mv_0^2 \) (where: \( \|v\| = v \))

16. Various works. Gravity: \( U_{x=0} = -W dy \) Springs: \( U_{x=0} = -\left(\frac{1}{2} k s_0^2 - \frac{1}{2} k s_1^2 \right) \)

17. Work and Energy: \( T_1 + U_{x=0} = T_2 \) where \( T = \frac{1}{2} mv^2 \)

18. Work Energy for a system: \( \sum_{\text{part.}} T_1 + \sum_{\text{part.}} U_{x=0} = \sum_{\text{part.}} T_2 \)

19. Form Equivalence
<table>
<thead>
<tr>
<th>Case</th>
<th>Statement</th>
<th>Mathematical implementation of the statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The work of a force is independent of the path</td>
<td>( \int_a^b F \cdot dr = -(V(b) - V(a)) )</td>
</tr>
<tr>
<td>2</td>
<td>A force is derivable from a potential function</td>
<td>( F = -\frac{dV(x)}{dx} )</td>
</tr>
<tr>
<td>3</td>
<td>A force conserves mechanical energy</td>
<td>( KE_1 + V_1 = KE_2 + V_2 )</td>
</tr>
</tbody>
</table>

20. Power: \( P = F \cdot v \)  
1 horsepower = 550 ft-lb/sec

21. Potentials \( G: \) \( V = G y \) \( S: \) \( V = \frac{1}{2} ks^2 \)

22. Impulse: \( \int_{t_1}^{t_2} F_1 dt \) (there are three: one for each direction)

23. Momentum: \( \int_{v_1}^{v_2} m dv = mv_2 - mv_1 \) (there are three: one for each direction)

24. Impulse/Momentum: Where, in general: \( I = \int_{t_1}^{t_2} F dt \)
\[ m(v_x)_1 + \sum_{x=1}^{x=2} I_{x=1-2} = m(v_x) \]
\[ m(v_y)_1 + \sum_{y=1}^{y=2} I_{y=1-2} = m(v_y) \]
\[ m(v_z)_1 + \sum_{z=1}^{z=2} I_{z=1-2} = m(v_z) \]

25. Conservation of Momentum: \( \sum_{i} m_i (v_{i})_1 = \sum_{i} m_i (v_{i})_2 \)
\[ (v_{i})_1 = (v_{i})_2 \]

26. Forms of restitution:
\[ e = \int R dt \]
\[ e = \frac{v - (v_g)_2}{(v_h)_1 - v} \]
\[ e = \frac{v - (v_a)_2}{(v_h)_1 - v} \]

27. Angular Momentum, Torque, Relationship: \( H = r \times mv \) \( M = r \times ma \) \( \dot{H} = M \) \( L = mv \)

28. Summary: \( \dot{H} = M \) \( L = F \)

29. Impulse Momentum (Angular and Linear):
\( (H_{\theta})_1 + \sum_{t_0}^{t_1} M_{\theta} dt = (H_{\theta})_2 \)
\( (L)_1 + \sum_{t_0}^{t_1} F dt = (L)_2 \)

30. \( \theta \) \( \omega = \frac{d\theta}{dt} \) \( \alpha = \frac{d\omega}{dt} \) \( \alpha \theta = \omega \theta \)

31. Constant:
\[ \omega = \omega_0 + \alpha_0 t \]
\[ \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_0 t^2 \]
\[ \omega^2 = \omega_0^2 + 2 \alpha_0 (\theta - \theta_0) \]

32. Radius of Gyration: \( I_G = mk_G^2 \) \( I_{\theta} = mk_\theta^2 \)

33. Parallel Axis Theorem: \( I = I_G + md^2 \)

34. Some formulas (your obligation to know the case):
\[ I = \frac{1}{12} m l^2 \]
\[ \dot{I} = \frac{1}{3} m l^2 \]
35. **Rotation about a fixed axis**

Velocity: \[ \mathbf{v} = \mathbf{\omega} \times \mathbf{r}_p \]

Acceleration: (component) \[ a_B = (a_B/A)_i + (a_B/A)_n \]

\[ a_i = \alpha \mathbf{r} \]

\[ a_n = \omega^2 r \]

(cross product 1) \[ \mathbf{a} = \mathbf{\alpha} \times \mathbf{r}_p + \mathbf{\omega} \times \mathbf{\omega} \times \mathbf{r}_p \]

(cross product 2) \[ \mathbf{a} = \mathbf{\alpha} \times \mathbf{r}_p - \omega^2 \mathbf{r}_p \]

35. **Motion of two points separated by a fixed distance**

Velocity: \[ \mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{BA} \]

Acceleration: (component) \[ a_B = a_A + (a_{BA/A})_i + (a_{BA/A})_n \]

\[ a_i = \alpha \mathbf{r} \quad a_n = \omega^2 r \]

(cross product 1) \[ a_B = a_A + \mathbf{\alpha} \times \mathbf{r}_{BA/A} + \mathbf{\omega} \times \mathbf{\omega} \times \mathbf{r}_{BA/A} \]

(cross product 2) \[ a_B = a_A + \mathbf{\alpha} \times \mathbf{r}_{BA/A} - \omega^2 \mathbf{r}_{BA/A} \]

36. **Motion of one point in two different frames**

Velocity: \[ \mathbf{v}_B = \mathbf{v}_A + (\mathbf{v}_{BA/A})_{xy} + \mathbf{\Omega} \times \mathbf{r}_{BA/A} \]

Acceleration:

\[ a_B = a_A + (a_{BA/A})_{xy} + 2\mathbf{\Omega} \times (\mathbf{v}_{BA/A})_{xy} + \mathbf{\dot{\Omega}} \times \mathbf{r}_{BA/A} + \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}_{BA/A} \]

37. **Kinetics of Bodies**

<table>
<thead>
<tr>
<th>ROW</th>
<th>Case</th>
<th>17.3 Linear Translation</th>
<th>17.4 Rotation about fixed axis</th>
<th>17.5 General</th>
</tr>
</thead>
</table>
| 1   | Force-direction1 | \[ \sum F_x = m(a_g)_x \]

\[ \sum F_n = m\omega^2 r_G \]

\[ \sum F_x = m(a_g)_x \]

| 2   | Force-direction2 | \[ \sum F_x = m(a_g)_x \]

\[ \sum F_y = m\alpha r_G \]

\[ \sum F_y = m(a_g)_y \]

| 3   | Moment about C.G. | \[ \sum M_G = 0 \]

\[ \sum M_G = I_G \alpha \]

| 4   | Moment about A DIFFERENT point. But still using G in kinematic part to account for offset | \[ \sum M_A = dm(a_g)_x \]

"d" is an offset |

\[ \sum M_G = r_G m(a_g)_x + I_G \alpha \]

\[ \sum M_G = (r_G^2 m + I_G) \alpha \]

\[ \sum M_G = I_G \alpha \]

\[ \sum M_K = I_K \alpha \]

\[ \sum M_K = (md^2 + I_G) \alpha \]

(this second one for rolling w/o slipping)