EXAMINATION IN: TOM028 Fluid Mechanics
CLASS: 11HMMT
DATE: December 3, 2014

NUMBER OF TASKS: FOUR
NUMBER OF PAGES: NINE (including this one)
APPENDIXES: NONE

EXAM AIDS: Calculator

TIME: 09:00 – 12:00 (3 hours)
LANGUAGE: English

ACADEMIC RESPONSIBLE
TEACHER(S): Thomas Impelluso
EXAMINER(S): 955 57 006

NOTES:

Po.box 7030, N-5020 Bergen. Phone: (+47) 55 58 75 00, Fax (+47) 55 58 77 90
Street address: Nygaardsgt. 112, Bergen
Problem 3.67: 25 points

A long, square wooden block is pivoted along one edge.

The block is in equilibrium when immersed in water to the depth shown.

There is no friction at the hinge.

Hint: shown on the right is a free body diagram you may wish to consider.

Hint: no friction means sum of moments is 0.0

Also

\[ I_{xx} = \frac{1}{12}WL^3 \]

1. List all equations that are to be used. For each and every equation: state how and why it will be used. This is to be separate from the solution. There must be a list of equations and a statement on each equation that is listed. (5 points)

2. Evaluate the density of the wood. (20 points total)

Problem 4.72: 25 points

A gate is 1m wide and 1.2 m tall and hinged at the bottom.

On the RIGHT side, the gate holds back a 1m deep body of water.

On the LEFT side, a 5cm diameter water jet hits the gate at a height of 1m.

1. List all equations that are to be used. For each and every equation: state how and why it will be used. This is to be separate from the solution. There must be a list of equations and a statement on each equation that is listed. (5 points)

2. Solve for the jet speed, \( V \), needed to hold the gate vertical. (20 points)
Problem 5.45: 25 points

Consider the following velocity field in the xy plane:

\[ \mathbf{v} = \left( \frac{Ax}{x^2 + y^2} \right) \mathbf{i} + \left( \frac{Ay}{x^2 + y^2} \right) \mathbf{j} \]

\[ A = 10 \text{m}^2 / \text{sec} \]

x and y are measured in meters.

NOTE: you will get an additional five points if your answers are clean, clear, organized, numbered and separated with lines (this is subjective). And those 5 points can make or break a grade. I am asking many questions here, so it will be an all or nothing 5 points based only on how clear your answers are to each question.

1. Is this an incompressible flow? Show why (2 points)
2. Derive an expression for the fluid acceleration (5 points)
3. Evaluate the velocity along the x axis (2 points)
4. Evaluate the velocity along the y axis (2 points)
5. Evaluate the acceleration along the x axis (2 points)
6. Evaluate the acceleration along the y axis (2 points)
7. Describe the fluid flow in words: what is happening? (5 points)

Problem 6.70: 25 points

A flow nozzle is a device for measuring the flow rate in a pipe. This particular nozzle is to be used to measure low speed air for which compressibility can be neglected.

During the operation, \( p_1 \) and \( p_2 \) (the pressures at the two points: 1 and 2) are recorded.

Assume the flow is frictionless.

Find the mass flow rate as a function of the difference in these two pressures.

Essentially, find \( k \) in the following

\[ m_{\text{flow, rate}} = k \sqrt{(p_1 - p_2)} \]

1. List all equations that are to be used. For each and every equation: state how and why it will be used. This is to be separate from the solution. There must be a list of equations and a statement on each equation that is listed. (5 points)
2. Show solution (20 points)
CHAPTER 3: STATICS

Pressure at a depth, h, in general

\[ p = \rho gh \]

Pressure on submerged surfaces

\[
\int_A y'dA = y_c A \\
\int_A x'dA = x_c A
\]

\[ p_c = (p_0 + \rho g y_c \sin \theta) \]

\[ F_{Re:all} = p_c A \]

\[ y' = y_c + \frac{\rho g \sin \theta}{F_R} (I_{xx}) \]

\[ x' = x_c + \frac{\rho g \sin \theta}{F_R} (I_{xy}) \]

Or

\[ y' = y_c + \frac{\rho g \sin \theta}{p_c A} (I_{xx}) \]

\[ x' = x_c + \frac{\rho g \sin \theta}{p_c A} (I_{xy}) \]

And, you can reform again if \( p_0 = 0 \)

\( I_{xx} \) and \( I_{xy} \) are the values of those second moments of inertia, but computed using a coordinate system located at the centroid

Geometric Background

\[
\int_A y^2 dA = I_{xx} \\
\int_A xydA = I_{xy}
\]

But these are for a specific axis. So, in general, use parallel axis

\[ I_{xx} = I_{xx'} + Ay^2 \] where y is the distance to the new axis.

\[ y' = y_c + \frac{\rho g \sin \theta}{p_c A} (I_{xx'}) \]

\[ p_c = \left(\rho g y_c \sin \theta\right) \quad \text{(this one assumes 0 ambient pressure)} \]

\[ y' = y_c + \frac{\rho g \sin \theta}{(\rho g y_c \sin \theta)A} (I_{xx'}) \]

\[ y' = y_c + \frac{1}{(y_c)A} (I_{xx'}) \]
CHAPTER 4: CONTROL VOLUME

Quantity that is flowing

\[ N_{\text{system}} = \int_{M_{\text{(system)}}} \eta p \, dm = \int_{V_{\text{(system)}}} \eta p \, dV \]

General Reynold's Transport Theorem

\[ \frac{dN_{\text{system}}}{dt} = \frac{\partial}{\partial t} \int_{V_{\text{(system)}}} \eta p \, dV + \int_{S_{\text{CS}}} \eta \rho \vec{V} \cdot d\vec{A} \]

CONSERVATION OF MASS becomes CONTINUITY EQUATION

\[ N = M \quad \eta = 1 \]

Definition

\[ M_{\text{system}} = \int_{M_{\text{(system)}}} \, dm = \int_{V_{\text{(system)}}} \rho \, dV \]

Law

\[ \frac{dM}{dt} \bigg|_{\text{System}} = 0 \]

Specific RTT

\[ \frac{dM}{dt} \bigg|_{\text{System}} = \frac{\partial}{\partial t} \int_{V_{\text{CV}}} \rho \, dV + \int_{S_{CS}} \rho \vec{V} \cdot d\vec{A} \]

Usage: \[ \frac{dM}{dt} \bigg|_{\text{System}} = 0 \]

CONSERVATION OF MOMENTUM becomes MOMENTUM EQUATION

\[ N = \bar{F} \quad \eta = \bar{V} \]

Definition

\[ \bar{F}_{\text{system}} = \int_{M_{\text{(system)}}} \vec{V} \, dm = \int_{V_{\text{(system)}}} \rho \vec{V} \, dV \]

Law

\[ \frac{d\bar{F}}{dt} \bigg|_{\text{System}} = \bar{F} \]

Specific RTT

\[ \frac{d\bar{F}}{dt} \bigg|_{\text{System}} = \frac{\partial}{\partial t} \int_{V_{\text{CV}}} \rho \vec{V} \, dV + \int_{S_{CS}} \rho \vec{V} \vec{V} \cdot d\vec{A} \]

Usage

\[ F_x = F_{S_x} + F_{B_x} = \frac{\partial}{\partial t} \int_{V_{\text{CV}}} u \rho \, dV + \int_{S_{CS}} u \rho \vec{V} \cdot d\vec{A} \]

\[ F_y = F_{S_y} + F_{B_y} = \frac{\partial}{\partial t} \int_{V_{\text{CV}}} v \rho \, dV + \int_{S_{CS}} v \rho \vec{V} \cdot d\vec{A} \]

\[ F_z = F_{S_z} + F_{B_z} = \frac{\partial}{\partial t} \int_{V_{\text{CV}}} w \rho \, dV + \int_{S_{CS}} w \rho \vec{V} \cdot d\vec{A} \]

**Bernoulli** is derived from the Momentum RTT but using:

1. Mass RTT
2. steady flow \[ \frac{p}{\rho} + gz + \frac{V_y^2}{2} = C \]
3. no flow across streamlines
4. incompressible flow
CHAPTER 5: DIFFERENTIAL ANALYSIS PART 1

\[ \nabla = \hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{e}_\phi \frac{\partial}{\partial \phi} \quad \nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \]

Conservation of Mass

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \quad \frac{1}{r} \frac{\partial (r \rho V_r)}{\partial r} + \frac{1}{r} \frac{\partial (r \rho V_\theta)}{\partial \theta} + \frac{\partial (\rho V_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \]

You should be able to reduce the above to steady and/or incompressible flow

Streamlines

\[ \frac{dx_s}{ds} \times \vec{V}(x_s) = 0 \quad \Rightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]

Combine above with Continuity Equation (for steady, incompressible flow) \( \nabla \cdot \vec{V} = 0 \) to get:

\[ u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x} \quad \text{V}_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \text{and} \quad \text{V}_\theta = -\frac{\partial \psi}{\partial r} \]

PSI is constant along a streamline. Q is found by difference of two streamline values (where you set the base \( = 0 \)) as you choose.

Behavior of Fluids

1. Linear Motion

\[ \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x} u + \frac{\partial \vec{V}}{\partial y} v + \frac{\partial \vec{V}}{\partial z} w \]

\[ \vec{a}_r = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial x} u + \frac{\partial \vec{V}}{\partial y} v + \frac{\partial \vec{V}}{\partial z} w \]

2. Linear Deformation (dilatation or NORMALIZED VOLUME RATE CHANGE)

\[ \frac{1}{V} \frac{dV}{dt} = \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = \nabla \cdot \vec{V} \]

3. Angular Motion

\[ \vec{\omega} = \frac{1}{2} \nabla \times \vec{V} \quad \vec{\omega} = \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \hat{i} + \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{j} + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k} \]

4. Angular Deformation

\[ \vec{\varepsilon} = \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \hat{i} + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \hat{j} + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \hat{k} \]
Conservation of Momentum

\[
\rho \frac{D\rho}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[ \mu \left( \frac{\partial u}{\partial x} - \frac{2}{3} \nabla \cdot \vec{V} \right) \right] + \frac{\partial}{\partial y} \left[ \mu \left( \frac{\partial v}{\partial y} - \frac{2}{3} \nabla \cdot \vec{V} \right) \right] + \frac{\partial}{\partial z} \left[ \mu \left( \frac{\partial w}{\partial z} - \frac{2}{3} \nabla \cdot \vec{V} \right) \right] + \frac{\partial}{\partial x} \left[ \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \right] + \frac{\partial}{\partial y} \left[ \mu \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \right] + \frac{\partial}{\partial z} \left[ \mu \left( \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} \right) \right]
\]

Euler's equation

\[
\rho \frac{D\vec{V}}{Dt} = \rho g - \nabla p
\]
CHAPTER 6 (DIFFERENTIAL ANALYSIS CONTINUED)

Euler in Cartesian

\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x} \]

\[ \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \rho g_y - \frac{\partial p}{\partial y} \]

\[ \rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z} \]

Euler in Cylindrical

\[ \rho \left( \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_r^2}{r} \right) = \rho g_r - \frac{\partial p}{\partial r} \]

\[ \rho \left( \frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} + \frac{V_\theta V_r}{r} \right) = \rho g_\theta - \frac{1}{r} \frac{\partial p}{\partial \theta} \]

\[ \rho \left( \frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z} \]

Euler in Streamline

\[ \rho \left( V_s \frac{\partial V_s}{\partial s} + \frac{\partial V_s}{\partial t} \right) = \frac{\partial p}{\partial s} - \rho g \frac{\partial z}{\partial s} \]

\[ \frac{V_s^2}{R} = \frac{1}{\rho} \frac{\partial p}{\partial n} + g \frac{\partial z}{\partial n} \]

Bernoulli

\[ \frac{p_1 + \frac{1}{2} g z_1}{\rho} = \frac{p_0 + \frac{1}{2} g z_0 + \frac{V_0^2}{2}}{\rho} \]
<table>
<thead>
<tr>
<th>Cartesian</th>
<th>Cylindrical</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What are the base vectors</strong></td>
<td></td>
</tr>
<tr>
<td>$\textbf{e}_x$, $\textbf{e}_y$, $\textbf{e}_z$</td>
<td>$\textbf{e}<em>r$, $\textbf{e}</em>\theta$, $\textbf{e}_z$</td>
</tr>
<tr>
<td><strong>What is the definition of length</strong></td>
<td></td>
</tr>
<tr>
<td>$ds^2 = dx^2 + dy^2 + dz^2$</td>
<td>$ds^2 = dr^2 + (r,d\theta)^2 + dz^2$</td>
</tr>
<tr>
<td><strong>How do base vectors change</strong></td>
<td></td>
</tr>
<tr>
<td>$\frac{de_x}{dx} = 0$, $\frac{de_y}{dy} = 0$, $\frac{de_z}{dz} = 0$</td>
<td>$\frac{de_r}{dr} = 0$, $\frac{de_\theta}{d\theta} = e_\theta$, $\frac{de_z}{dz} = 0$</td>
</tr>
<tr>
<td><strong>What does the nabla look like</strong></td>
<td></td>
</tr>
<tr>
<td>$\nabla = e_x \frac{\partial}{\partial x} + e_y \frac{\partial}{\partial y} + e_z \frac{\partial}{\partial z}$</td>
<td>$\nabla = e_r \frac{\partial}{\partial r} + e_\theta \frac{\partial}{\partial \theta} + e_z \frac{\partial}{\partial z}$</td>
</tr>
<tr>
<td><strong>GRADIENT</strong></td>
<td><strong>What does nabla do to a function</strong></td>
</tr>
<tr>
<td>$\nabla f = e_x \frac{\partial f}{\partial x} + e_y \frac{\partial f}{\partial y} + e_z \frac{\partial f}{\partial z}$</td>
<td>$\nabla = e_r \frac{\partial V_r}{\partial r} + e_\theta \frac{\partial V_\theta}{\partial \theta} + e_z \frac{\partial V_z}{\partial z}$</td>
</tr>
<tr>
<td><strong>DIVERGENCE</strong></td>
<td>$\nabla \cdot \textbf{V} = \left( \frac{\partial V_r}{\partial r} + \frac{\partial V_\theta}{r \partial \theta} + \frac{\partial V_z}{\partial z} \right)$</td>
</tr>
<tr>
<td><strong>CURL</strong></td>
<td><strong>What does nabla do to a vector using cross product</strong></td>
</tr>
<tr>
<td>$\nabla \times \textbf{V} = \left( \begin{array}{c} \frac{\partial V_z}{\partial y} - \frac{\partial V_\theta}{\partial z} \ \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \ \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \end{array} \right) \textbf{e}_x$</td>
<td>$\nabla \times \textbf{V} = \left( \begin{array}{c} \frac{1}{r} \frac{\partial V_\theta}{\partial \theta} - \frac{\partial V_r}{\partial z} \ \frac{\partial V_r}{\partial r} - \frac{\partial V_\theta}{\partial r} \ \frac{1}{r} \frac{\partial V_r}{\partial \theta} - \frac{\partial V_\theta}{\partial \theta} \end{array} \right) \textbf{e}_z$</td>
</tr>
</tbody>
</table>